Unified Aeroelastic Flutter Theory for Very Low Aspect Ratio Panels

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A theory unifying the flutter analyses of orthotropic and isotropic panels having very low aspect ratios and exposed to an inviscid potential flow on the upper surfaces is developed. The analysis considers an infinitely long panel of finite width, simply supported on the side edges, and resting on a spring foundation. Three-dimensional linearized aerodynamic and classical plate theories are used. Traveling wave modes are utilized as solutions to the panel's aeroelastic equation of motion. The resulting aerodynamic integral is evaluated approximately using both numerical and analytical methods. The structural and aerodynamic generic variables, which are instrumental in establishing this unified theory, are derived using affine transformations. Subsonic as well as supersonic flutter and divergence boundaries are determined. The effects of air to panel mass ratios and midplane forces are examined. Viscous structural damping is found to be destabilizing. There is a general agreement between the results of this analysis and those obtained for isotropic panels by previous investigators.

Nomenclature

a_0	= finite panel length in affine space
b,b_0	= panel width in Cartesian and affine space, respectively
c,c_0	= wave speed in Cartesian and affine space, respectively
c_r	= reference wave speed
D^*,D_{ij}	= generalized rigidity ratio and elastic parameters, respectively
G_s,g_s	= structural damping and structural damping coefficient, respectively
h	= panel thickness
$I(\eta)$	= aerodynamic integral
i	$=\sqrt{-1}$
K	= spring constant
k_0,k_{0y}	= generic buckling coefficients
k_s	= spring parameter
ℓ_0,M	= wavelength and Mach number, respectively
$(N_x,N_y),(N_{x0},N_{y0})$	= midplane stresses (+ in compression) in Cartesian and affine space, respectively
P,P_0	= pressure in Cartesian and affine space, respectively
t	= time
U,U_0	= airflow speed in Cartesian and affine space, respectively
W,w	= panel deflections
$(x,y,z),(x_0,y_0,z_0)$	= Cartesian and affine coordinates, respectively
α,α_0	= speed of sound in Cartesian and affine space, respectively
λ	= dynamic pressure
μ	= mass ratio
ρ,ρ_∞	= panel mass density and air mass density, respectively
τ	= time in affine space
ϕ	= velocity potential
ω_0,ω_r	= frequency and reference frequency, respectively

Superscript

$(\ )^*$	= nondimensionalized generic variables

Subscript

cr	= critical or flutter quantities

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Introduction

ALTHOUGH panel flutter is generally believed to be a supersonic phenomenon, its existence at subsonic speeds has been established by some investigators. Miles' early analysis[1] for infinitely long and infinitely wide panels was supplemented by later analysts such as Dugundji et al.[2] and Dowell,[3] who considered infinitely long but finitely wide panels. While Refs. 1-3 used three-dimensional linearized aerodynamics, Dugundji[4] and Oyibo[5] used the aerodynamic piston theory. A primary objective of Refs. 1-5 is to construct asymptotic flutter boundaries for low aspect ratio panels (finite dimensions) using panels of infinite dimensions which are more tractable mathematically—the tradeoff being the possibility of obtaining simple algebraic expressions as the end results. In an attempt to unify the subsonic and supersonic analyses, Dowell[3] discovered some discrepancies between his results and those of Bohon and Dixon[6] and Cunningham[7] (who used more elaborate methods) in the high supersonic Mach number range. This problem was seemingly resolved by Dowell[8] when he concluded, with the help of the studies by Spriggs et al.,[9] that the flutter modes used in Ref. 3 were inadequate in the high supersonic region.[10] These studies agreed that, while divergence is possible, traveling-wave type flutter is the principal instability at subsonic speeds.

The analytical complexities encountered on including the orthotropic properties of the panels appear to be a principal reason why most of the analyses have been restricted to membranes and isotropic panels. However, the increasing importance of nonisotropic materials in aerospace applications make it more desirable than ever to maximize the physical understanding of anisotropic problems. Brunelle[11-14] and Oyibo[5,14-17] have shown that affine transformation is a tool that makes this possible.

In this paper, the generic structural and aerodynamic variables derived by affinely transforming the aeroelastic equations of motion for an infinitely long orthotropic panel of finite width on a spring foundation are utilized in the flutter analysis. Consequently, the end results represent unified asymptotic flutter solutions for very low aspect ratio panels, since these variables incorporate both the orthotropic and isotropic properties.

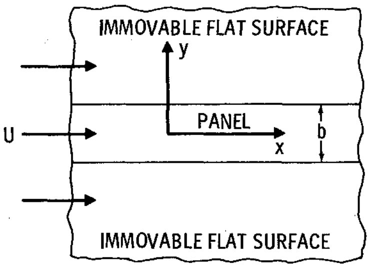

Fig. 1 An infinitely long panel of finite width having viscous structural damping and resting on a spring foundation.

Statement of Problem

Consider an infinitely long flat orthotropic panel of width b and a uniform thickness h shown in Fig. 1. The panel is hinged at the side edges, subjected to an inviscid potential flow on the top surface and midplane force intensities N_x and N_y. It has a viscous structural damping G_s (N-s m^{-3}) and rests on a spring foundation K (N-m^{-3}).

Aeroelastic Equation of Motion

Utilizing three-dimensional linearized aerodynamic and classical plate theory, the aeroelastic equation of motion for the panel in the physical space is

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+N_x\frac{\partial^2 w}{\partial x^2}$$
$$+N_y\frac{\partial^2 w}{\partial y^2}+G_s\frac{\partial w}{\partial t}+\rho h\frac{\partial^2 w}{\partial t^2}+Kw+P=0 \qquad (1)$$

where, from Bernoulli's equation, the pressure P is given by

$$P=-\rho_\infty\left[\frac{\partial \phi}{\partial t}+U\frac{\partial \phi}{\partial x}\right]_{z=0} \qquad (2)$$

and where ϕ, the velocity potential, satisfies the linear perturbation equation (convected wave equation) given by

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-\frac{1}{\alpha^2}\left[\frac{\partial^2\phi}{\partial t^2}+2U\frac{\partial^2\phi}{\partial x\partial t}+U^2\frac{\partial^2\phi}{\partial x^2}\right]=0 \qquad (3)$$

The boundary conditions for the simply supported side edges on the panel deflection are

$$w\left(x,\frac{b}{2},t\right)=w\left(x,\frac{-b}{2},t\right)=\frac{\partial^2 w}{\partial y^2}\left(x,\frac{b}{2},t\right)=\frac{\partial^2 w}{\partial y^2}\left(x,\frac{-b}{2},t\right)=0$$
$$w(\infty,y,t) \text{ and } w(-\infty,y,t) \text{ finite} \qquad (4)$$

The boundary conditions on the velocity potential are

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=0}=\frac{\partial w}{\partial t}+U\frac{\partial w}{\partial x}, \qquad -b/2\le y\le b/2$$
$$=0, \qquad /y/>b/2 \qquad (5)$$

$\phi(\infty,y,z,t)$ and $\phi(-\infty,y,z,t)$ must be bounded, $\phi(x,y,\infty,t)$ and $\phi(x,y,-\infty,t)$ must be finite and satisfy Summerfeld's radiation conditions.[18,19]

Affine Transformations and the Generic Variables

Consider the affine transformations

$$x=(D_{11})^{1/4}x_0, \qquad y=(D_{22})^{1/4}y_0, \qquad t=(\rho h b_0^4/\pi^4)^{1/2}\tau \qquad (6)$$

Equations (6) map Eqs. (1-3) into the affine space to produce the following:

$$\frac{\partial^4 w}{\partial x_0^4}+2D^*\frac{\partial^4 w}{\partial x_0^2\partial y_0^2}+\frac{\partial^4 w}{\partial y_0^4}+N_{x0}\frac{\partial^2 w}{\partial x_0^2}+N_{y0}\frac{\partial^2 w}{\partial y_0^2}$$
$$+G_s\omega_r\frac{\partial w}{\partial \tau}+\left(\frac{\pi}{b_0}\right)^4\frac{\partial^2 w}{\partial \tau^2}+Kw+P_0(x_0,y_0,z,t)=0 \qquad (7)$$

where

$$P_0=-\rho_\infty\left[\omega_r\frac{\partial\phi}{\partial\tau}+U_0\frac{\partial\phi}{\partial x_0}\right]_{z=0} \qquad (8)$$

and

$$\frac{\partial^2\phi}{\partial x_0^2}+(D_{11}/D_{22})^{1/2}\frac{\partial^2\phi}{\partial y_0^2}+(D_{11})^{1/2}\frac{\partial^2\phi}{\partial z^2}$$
$$-\frac{1}{\alpha_0^2}\left[\omega_r^2\frac{\partial^2\phi}{\partial\tau^2}+2\omega_r U_0\frac{\partial^2\phi}{\partial x_0\partial_\tau}+U_0^2\frac{\partial^2\phi}{\partial x_0^2}\right]=0 \qquad (9)$$

where

$$D^*\triangleq\frac{D_{12}+2D_{66}}{(D_{11}D_{22})^{1/2}}, \qquad N_{x0}\triangleq\frac{N_x}{(D_{11})^{1/2}}, \qquad N_{y0}\triangleq\frac{N_y}{(D_{22})^{1/2}}$$
$$U_0\triangleq\frac{U}{(D_{11})^{1/4}}, \qquad \alpha_0\triangleq\frac{\alpha}{(D_{11})^{1/4}}, \qquad \omega_r\triangleq\left(\frac{\pi^4}{\rho h b_0^4}\right)^{1/2} \qquad (10)$$

Equations (7-9) are the panel's aeroelastic equation of motion, the aerodynamic pressure equation, and the linearized, perturbed potential equation, respectively, in the affine space. Equations (10) define some of the generic variables. The generic variable D^*, called the generalized rigidity ratio, is predicted to be in a closed interval $0\le D^*\le 1$, for all orthotropic panels (for isotropic panels, $D^*=1$).[11-17]

Solution of the Partial Differential Equation

In order to carry out the flutter analysis, it is necessary to solve the eigenvalue problem defined by Eqs. (7-9) subject to the corresponding boundary conditions [Eqs. (4) and (5) in affine space]. Consequently, the panel modes are assumed to be traveling waves given by

$$w=\left[\sum_{n=1,3,\ldots}\left(W_n\cos\frac{n\pi y_0}{b_0}\right)\right.$$
$$\left.+\sum_{m=2,4,\ldots}\left(W_m\sin\frac{m\pi y_0}{b_0}\right)\right]\exp\left[\frac{2\pi i}{\ell_0}(c\tau-x_0)\right] \qquad (11)$$

It is further assumed that using only the first term of Eq. (11) in this analysis should introduce very small error in the lowest flutter speed. Hence the panel mode shape and the corresponding velocity potential are, respectively, given by

$$w=W\cos\left(\frac{\pi y_0}{b_0}\right)\exp\left[\frac{2\pi i}{\ell_0}(c\tau-x_0)\right] \qquad (12)$$

and

$$\phi=\Phi(y_0,z)\exp\left[\frac{2\pi i}{\ell_0}(c\tau-x_0)\right] \qquad (13)$$

To obtain the expression for P_0 [Eq. (8)], it is necessary to solve for ϕ in Eq. (9). Therefore, using the "separation of variables" method, ϕ is assumed to be of the following form

$$\Phi(y_0,z) = Y(y_0)Z(z) \tag{14}$$

Define

$$\frac{Y''}{Y} = -u_0^2; \qquad \left(Y'' = \frac{\partial^2 Y}{\partial y_0^2}\right) \tag{15}$$

When Eqs. (13-15) and the corresponding boundary conditions are substituted into Eq. (9), and use is made of Fourier transform methods, the solution for Φ is given by

$$\Phi(y_0,z) = 2\left(\frac{2\pi i}{\ell_0}\right)(U_0 - c_0)W \times \int_0^\infty e^{-(\ )z} \frac{\cos(u_0 b_0/2)\cos(u_0 y_0)\,du_0}{(\ )[(\pi/b_0)^2 - u_0^2]} \tag{16}$$

where

$$(\ )^2 = \frac{u_0^2}{(D_{22})^{1/2}} + \frac{1}{(D_{11})^{1/2}}\left(\frac{2\pi}{\ell_0}\right)^2\left[1 - M^2\left(\frac{1-c_0}{U_0}\right)^2\right] \tag{17}$$

and $0 \le M(1 - c_0/U_0) < 1$ (1 is a singular point).

Substituting Eqs. (13), (16), and (17) into Eq. (8), the expression for the pressure in the affine space (P_0) is obtained. By further substituting this expression and Eq. (12) into Eq. (7), an equation results which can be solved by Galerkin's method. Thus, multiplying both sides of this equation by $\cos\pi y_0/b_0$ and integrating over the span, the Galerkin nontrivial solution, upon nondimensionalizing, reduces (in several steps) to

$$c_0^{*2} - \frac{ig_s}{4}\left(\frac{\ell_0}{b_0}\right)c_0^* - T^2 + \mu(U_0^* - c_0^*)^2 I(\eta) = 0 \tag{18}$$

where

$$T^2 = \frac{1}{4}\left[\left(\frac{\ell_0}{2b_0}\right)^2 - (k_0 - 2D^*) + \left(\frac{2b_0}{\ell_0}\right)^2 + \left(\frac{\ell_0}{2b_0}\right)^2 k_s - \left(\frac{\ell_0}{2b_0}\right)^2\left(\frac{b_0}{a_0}\right)^2 k_{0y}\right]$$

$$k_0 = \frac{N_{x0}b_0^2}{\pi^2}, \qquad k_s = \frac{Kb_0^4}{\pi^4}, \qquad k_{0y} = \frac{N_{y0}a_0^2}{\pi^2} \tag{19}$$

$$I(\eta) = \frac{\pi}{2}\int_0^\infty \frac{\cos^2\sigma\, d\sigma}{(\sigma^2+\eta)^{1/2}[(\pi/2)^2 - \sigma^2]^2}$$

$$\sigma = \frac{u_0 b_0}{2}, \qquad \eta = \left(\frac{D_{22}}{D_{11}}\right)^{1/2}\left(\frac{b_0\pi}{\ell_0}\right)^2\left[1 - M^2\left(\frac{1-c_0}{U_0}\right)^2\right] \tag{20}$$

where $(D_{22}/D_{11})^{1/2}$ is always positive and not equal to zero (zero is a singular point).

$$U_0^* = U_0/c_r, \qquad c_0^* = c_0/c_r, \qquad c_0 = c\omega_r, \qquad c_r = 2\omega_r b_0/\pi \tag{21}$$

$$g_s = \frac{G_s b_0^2}{\pi^2(\rho h)^{1/2}}, \qquad \mu = \frac{\rho_\infty (D_{22})^{1/4} b_0}{\rho h} \tag{22}$$

Equations (19-22) define another set of generic variables and $I(\eta)$ is the aerodynamic integral. Equation (18) is rearranged as follows

$$c_0^{*2} - [1/(1+\mu I)][ig_s(\ell_0/4b_0) + 2\mu U_0^* I]c_0^* - T^2/(1+\mu I) + (\mu U_0^{*2} I)/(1+\mu I) = 0 \tag{23}$$

Equation (23) is the characteristic equation from which the eigenvalues of c_0^* are extracted for the flutter analysis. However, it is difficult to explicitly solve for c_0^* in Eq. (23) since I is generally a complicated function of c_0^*.

Flutter Analysis

To determine the flutter boundaries, first, the eigenvalues of c_0^* in Eq. (23) are calculated for various combinations of panel and flight parameters. On using these eigenvalues, the most critical dynamic pressure (or velocity), that satisfies a certain defined condition on the damping g_s, is found and defines flutter.[20] However, if $g_s = 0$, flutter is characterized by a coalescence of two eigenvalues.

Incompressible Analysis

In the flight speed range where compressibility effects are negligible, $M = 0$, and from Eq. (20), I becomes independent of c_0^*. Equation (23) therefore becomes quadratic in c_0^*, with the solution given by

$$c_0^* = 1/(1+\mu I)\{ig_s\ell_0/8b_0 + \mu U_0^* I \pm [(1+\mu I)T^2 - (g_s\ell_0/8b_0)^2 - \mu U_0^{*2} I + i(g_s\ell_0/4b_0)\mu U_0^* I]^{1/2}\} \tag{24}$$

Equation (24) gives an explicit expression for the eigenvalues in terms of the generic variables. Flutter conditions can now be established for undamped and damped systems in incompressible fluids.

Undamped Systems

For undamped systems, $g_s = 0$. Hence, Eq. (24) reduces to

$$c_0^* = 1/(1+\mu I)\{\mu U_0^* I \pm [(1+\mu I)T^2 - \mu U_0^{*2} I]^{1/2}\} \tag{25}$$

Flutter sets in when the quantity under the radical becomes negative (for c_0^* becomes complex). This occurs when

$$\mu U_0^{*2} I \ge (1+\mu I)T^2 \tag{26}$$

$$\lambda \ge T^2(1+\mu I)/I \tag{27}$$

where the nondimensionalized generic dynamic pressure λ is defined as

$$\lambda = \mu U_0^{*2} \tag{28}$$

Also, from Eq. (26)

$$U_0^* \ge T[(1+\mu I)/\mu I]^{1/2} \tag{29}$$

The minima of Eqs. (27) and (29) therefore define the flutter boundaries.

Damped Systems

For damped systems, $g_s > 0$. Flutter oscillations are characterized by negative imaginary parts of the eigenvalues c_0^* in Eq. (24). That is,

$$c_2^* < 0 \tag{30}$$

where

$$c_0^* \equiv c_1^* + ic_2^* \tag{31}$$

Routine algebra reduces the condition in Eq. (30) to

$$g_s/2 \leq \left(\frac{2b_0}{\ell_0}\right)[+2(u^2+v^2)^{1/2}-2u]^{1/2} \tag{32}$$

where

$$u=(I+\mu I)T^2-(g_s\ell_0/8b_0)^2-\mu U_0^{*2}I$$

$$v=g_s(\ell_0/4b_0)\mu U_0^* I$$

Equation (32) therefore gives the constraint on the damping for flutter. On further simplification, Eq. (32) reduces to

$$\mu U_0^{*2} \geq T^2/I \tag{34}$$

or

$$\lambda \geq T^2/I \tag{35}$$

and

$$U_0^* \geq T/(\mu I)^{1/2} \tag{36}$$

The minima of Eqs. (35) and (36) define the flutter boundaries for the damped systems. The flutter quantities are independent of damping. A check of Eq. (24) reveals that at the onset of instability $c_0^* \equiv 0$. This suggests that the instability in this case is divergence.

In order to obtain the numerical values of these flutter quantities, the aerodynamic integral, I, has to be evaluated. However, since it appears I cannot be integrated in terms of known functions, it has been evaluated numerically for the most part. For real positive values of η a "trial and error" method yielded the following expressions (which are in agreement with the numerical results for I)

$$\begin{aligned} I &= 0.3384-0.2800\log_{10}\eta && 0.01\leq\eta\leq 1.0 \\ &= 0.3903-0.3492[\eta/(\eta+1)]^{2.75} && 1.0\leq\eta\leq 100 \\ &= 0.3384\{1-[\eta/(\eta+1)]^{16.21}\} && \eta\geq 100 \end{aligned} \tag{37}$$

Compressible Analysis

In the flight speed range where compressibility effects are important ($M \neq 0$), the analysis is basically the same as the incompressible case except that c_0^* cannot be solved for explicitly from Eq. (23). For undamped systems, the flutter boundaries are determined by seeking eigenvalues of c_0^* which are simultaneously the zeros of Eq. (23) and its derivative with respect to c_0^*. This procedure uses Eq. (37) since the conclusion from Eqs. (17) and (20) is that the linearized aerodynamic theory only permits subsonic disturbances in the flowfield ($\eta > 0$) for subcritical generic variables. The divergence boundaries are determined by the minima of Eq. (24) with $c_0^* \equiv 0$ [see Eqs. (35) and (36)].

The Generic Variables and the Instability Boundaries

Figures 2-14 show the effects of the various generic variables on the instability boundaries for $N_y = 0$. While Figs. 2-9 depict the results for incompressible flow, Figs. 10-14 compare the compressible and incompressible analyses. Figures 2-4 basically illustrate the effects of the quantity $(k_0 - 2D^*)$, which combines the material property and the midplane forces, air to panel mass ratio and D_{22}/D_{11}. Figures 5-7 show a similar situation for the spring foundation parameter. In both cases the divergence boundaries are found to be the limit as μ approaches zero. The critical static value for $(k_0 - 2D^*)$ is 2[14,15] (confirmed by Figs. 2-4). λ_{cr} varies almost linearly with $(k_0 - 2D^*)$. It is also seen that λ_{cr} increases with increasing (D_{22}/D_{11}) (this ratio has a semiclosed

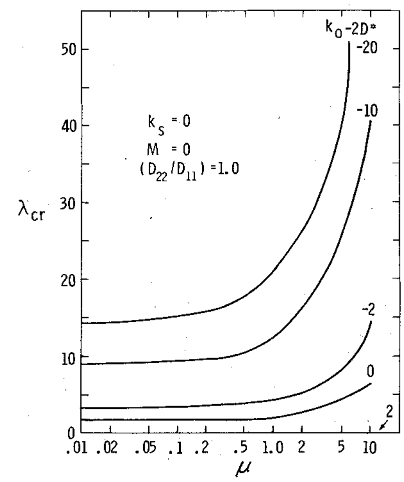

Fig. 2 Flutter dynamic pressure vs mass ratio for various $(k_0 - 2D^*)$.

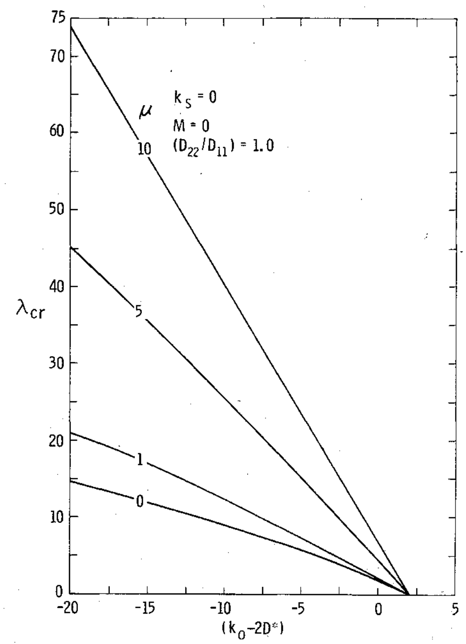

Fig. 3 Flutter dynamic pressure vs $(k_0 - 2D^*)$ for various mass ratios.

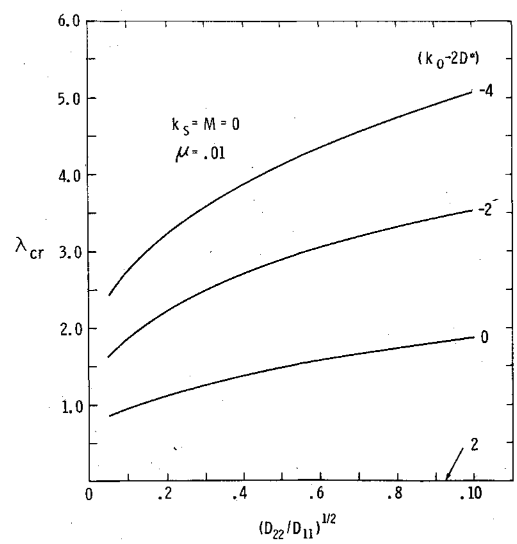

Fig. 4 Flutter dynamic pressure vs $(D_{22}/D_{11})^{1/2}$ for various $(k_0 - 2D^*)$.

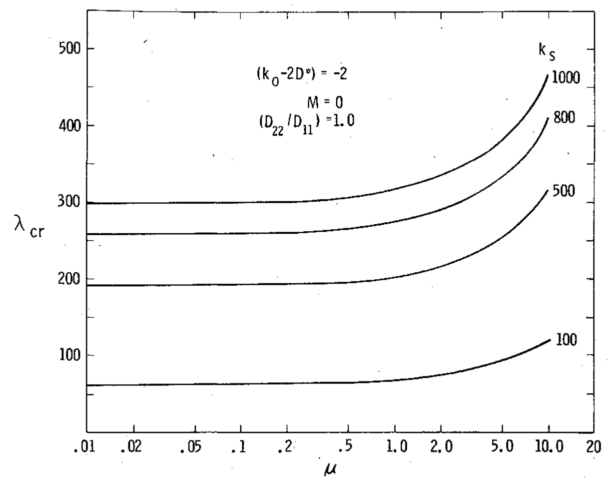

Fig. 5 Flutter dynamic pressure vs mass ratio for various spring constants.

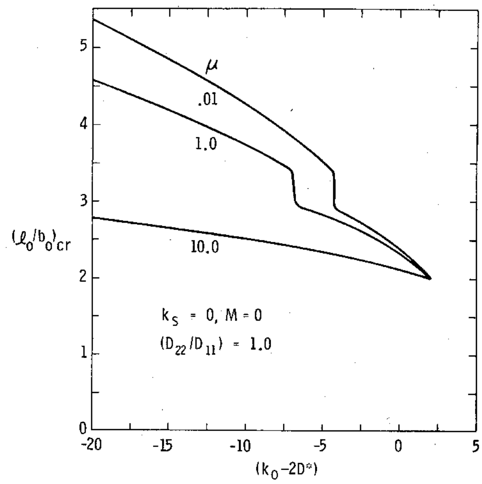

Fig. 8 Flutter wavelength vs $(k_0 - 2D^*)$ for various mass ratios.

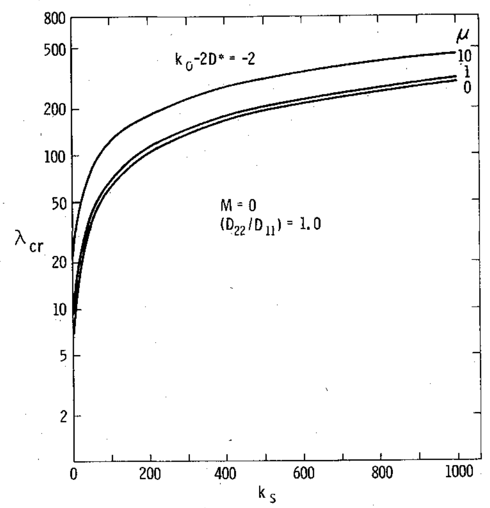

Fig. 6 Flutter dynamic pressure vs spring constants for various mass ratios.

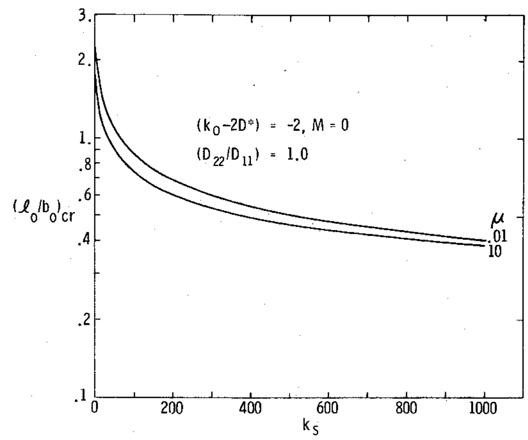

Fig. 9 Flutter wavelength vs spring constants for various mass ratios.

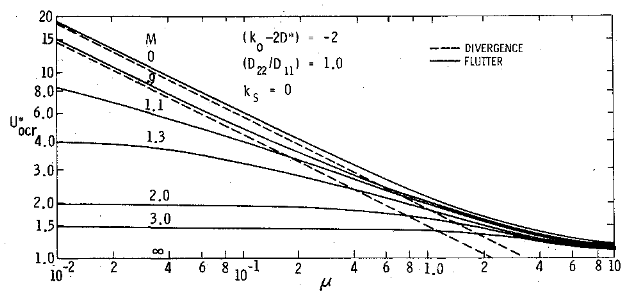

Fig. 10 Flutter velocity vs mass ratio for various Mach numbers.

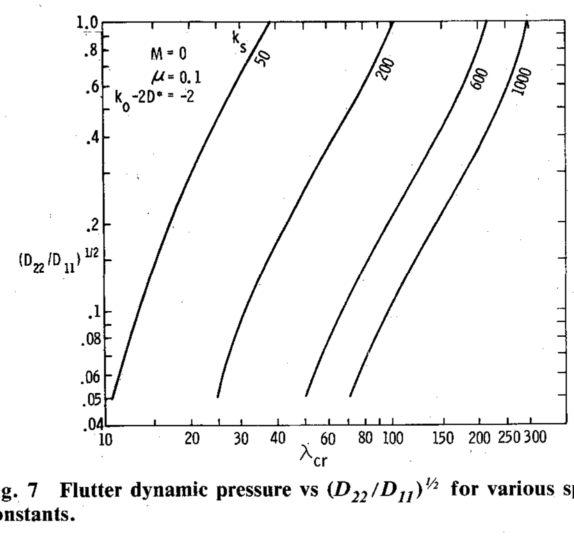

Fig. 7 Flutter dynamic pressure vs $(D_{22}/D_{11})^{1/2}$ for various spring constants.

interval, $0<D_{22}/D_{11}\leq 1.0$, for all orthotropic panels). Figures 8 and 9 display the variation of the critical wavelength with (k_0-2D^*) and k_s (the spring parameter), respectively. It is not clear why there are "bumps" on the graphs for low μ in Fig. 8. However, it is seen from these figures that $(\ell_0/b_0)_{cr}$ varies considerably with k_s and (k_0-2D^*).

Figure 10 indicates that while flutter could occur at any Mach number, divergence is essentially a subsonic phenomenon. For supersonic Mach numbers damped systems exhibit flutter (not divergence). It is also seen that damping is destabilizing. The trends predicted by Figs. 10 and 11 (also seen in Figs. 2 and 5) are that flutter is: 1) a constant dynamic pressure phenomenon at subsonic speeds and small μ, 2) a constant speed phenomenon at high supersonic speeds or large μ, and 3) independent of Mach numbers for large μ. Figures 13 and 12 show the flutter wave speed and frequency [defined by $\omega_0^* = 2c_0^*/(\ell_0/b_0)$]. It is seen that for $M<1$, $c_{0_{cr}}^*$ and $\omega_{0_{cr}}^*$ approach zero as $\mu \rightarrow 0$ (divergence). Figure 14 indicates that the flutter wavelength is on the order of 2 for the values of (k_0-2D^*) and k_s chosen. However, for other values of these quantities the flutter wavelength could have a fairly considerable variation (see Figs. 8 and 9).

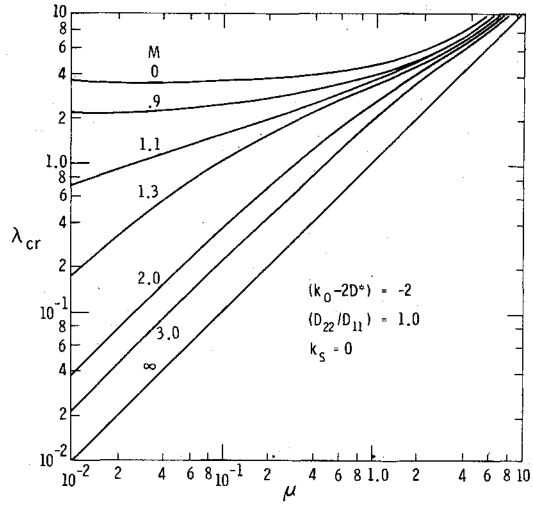

Fig. 11 Flutter dynamic pressure vs mass ratio for various Mach numbers.

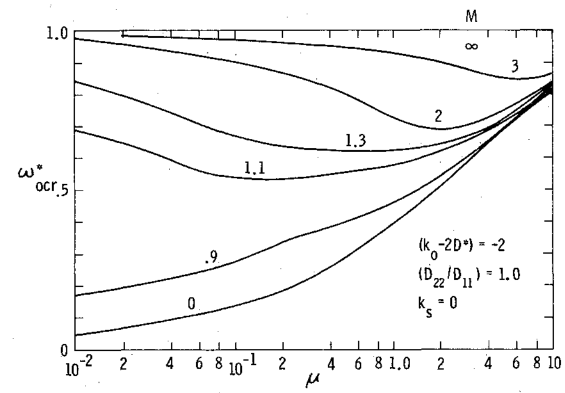

Fig. 12 Flutter frequency vs mass ratio for various Mach numbers.

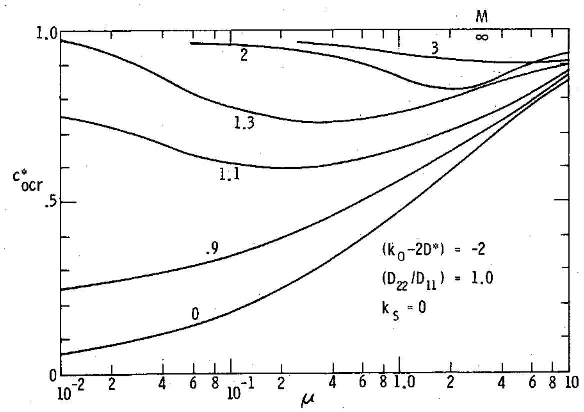

Fig. 13 Flutter wavespeed vs mass ratio for various Mach numbers.

Concluding Remarks

This paper has attempted to construct a unified asymptotic flutter theory for very low aspect ratio orthotropic and isotropic panels. The analysis considers an infinitely long panel of finite width exposed to an inviscid potential flow on the top surface and uses three-dimensional linearized aerodynamic, classical plate and traveling wave theories.

A key tool in the theory formulation is the affine transformation. Velocity potential is a function of the elastic constant ratios $(D_{22}/D_{11})^{1/2}$ in the affine space. The resulting added singularity is removed by restricting $(D_{22}/D_{11})^{1/2}$ to positive nonzero values (a zero value really corresponds to a

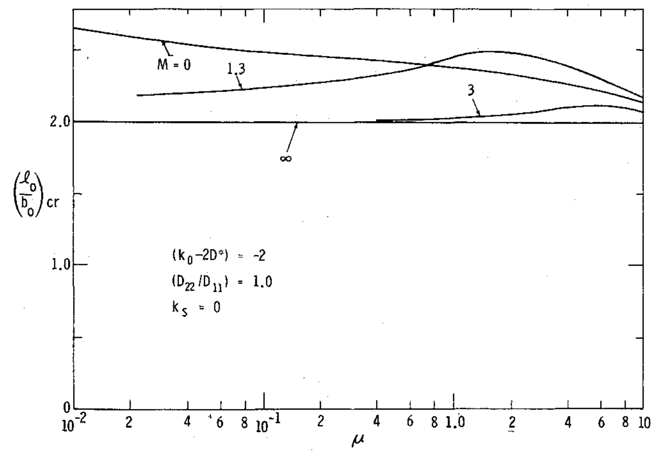

Fig. 14 Flutter wavelength vs mass ratio for various Mach numbers.

one-dimensional problem). The other singularity is when the effective Mach number, $M(1-c_0/U_0)$, equals one (removable by using the nonlinear aerodynamic theory). The mode shapes used are not very accurate at high supersonic speeds.[8]

In spite of these drawbacks, the present unified theory seems to enhance the physical understanding of the problem significantly, allowing the following conclusions to be made: Flutter is: 1) a constant dynamic pressure phenomenon at subsonic speeds and small μ, 2) a constant speed phenomenon at high supersonic speeds or large μ, and 3) independent of Mach numbers for large μ. Viscous structural damping is destabilizing.

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